To Actual Definiton From Historical Definition Of The Gamma Function

By Abdullah Aydemir, 18.12.2022

Definition (Euler, 1730):

Let $x > 0$

$$\Gamma(x) = \int_0^1 (-logt)^{x-1}\, dt$$

where $logx$ is lnx.

Theorem:

For $x > 0$

$$\Gamma(x) = \int_0^\infty t^{x-1}\, e^{-t} dt\,.$$

Proof:

Consider for $x > 0$

$$\Gamma(x) = \int_0^1 (-logt)^{x-1}\, dt.$$

Let $u = -logt$, then

$$t = e^{-u} \qquad du = -\frac{dt}{t}$$

$$t = 0 \rightarrow u = \infty \quad t = 1 \rightarrow u = 0$$

$$\Gamma(x) = \int_\infty^0 u^{x-1}\,(-e^{-u})du$$

$$\Gamma(x) = \int_0^\infty u^{x-1}\, e^{-u} du$$

$$\Gamma(x) = \int_0^\infty t^{x-1}\, e^{-t} dt$$

■