A property of the Gamma function-By **Abdullah Aydemir,** 15.12.2022

For $n \in \mathbb{Z}^+$ and $x \notin \mathbb{Z}^- \cup \{0\}$, we have

$$\Gamma(x) = \frac{\Gamma(x+n)}{x(x+1)(x+2)\dots(x+n-2)(x+n-1)}.$$

Proof:

$$\Gamma(x+1) = x.\Gamma(x) \implies \Gamma(x) = \frac{\Gamma(x+1)}{x}, x \neq 0$$

$$\Gamma(x+2) = \Gamma\big((x+1)+1\big) = (x+1)\Gamma(x+1) = (x+1)x\Gamma(x)$$

$$\implies \Gamma(x) = \frac{\Gamma(x+2)}{x(x+1)}, x \neq 0, -1$$

$$\Gamma(x+3) = \Gamma\big((x+2)+1\big) = (x+2)\Gamma(x+2) = (x+2)(x+1)x\Gamma(x)$$

$$\implies \Gamma(x) = \frac{\Gamma(x+3)}{x(x+1)(x+2)}, x \neq 0, -1, -2$$

$$\dots$$

$$\Gamma(x+n) = \Gamma\big((x+n-1)+1\big) = (x+n-1)\Gamma(x+n-1)$$
$$= (x+n-1)(x+n-2)\dots(x+2)(x+1)x\Gamma(x)$$

$$\implies \Gamma(x) = \frac{\Gamma(x+n)}{x(x+1)(x+2)\dots(x+n-2)(x+n-1)}, x \neq 0, -1, -2, \dots$$

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