A property of the Gamma function:

For $n \in \mathbb{Z}^+$,

$$\Gamma\left(\frac{2n+1}{2}\right) = \Gamma\left(n+\frac{1}{2}\right) = \frac{1.3.5\ldots(2n-1)}{2^n}\sqrt{\pi}.$$

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Proof:

$$\Gamma(x+1) = x.\Gamma(x)$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

Consider for $k \in \mathbb{Z}^+$, $\Gamma\left(\frac{2k+1}{2}\right) = \Gamma\left(k+\frac{1}{2}\right)$.

$$k = 1 \Longrightarrow \Gamma\left(\frac{3}{2}\right) = \Gamma\left(\frac{1}{2}+1\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{\pi}$$

$$k = 2 \Longrightarrow \Gamma\left(\frac{5}{2}\right) = \Gamma\left(\frac{3}{2}+1\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{3}{2}\frac{1}{2}\sqrt{\pi} = \frac{1.3}{2^2}\sqrt{\pi}$$

$$k = 3 \Longrightarrow \Gamma\left(\frac{7}{2}\right) = \Gamma\left(\frac{5}{2}+1\right) = \frac{5}{2}\Gamma\left(\frac{5}{2}\right) = \frac{5}{2}\frac{3}{2}\frac{1}{2}\sqrt{\pi} = \frac{1.3.5}{2^3}\sqrt{\pi}$$

…

$$k = n \Longrightarrow \Gamma\left(\frac{2n+1}{2}\right) = \Gamma\left(\frac{2n-1}{2}+1\right) = \frac{(2n-1)}{2}\Gamma\left(\frac{2n-1}{2}\right)$$

$$= \frac{(2n-1)}{2}\frac{(2n-3)}{2}\ldots\frac{5}{2}\frac{3}{2}\frac{1}{2}\sqrt{\pi} = \frac{1.3.5\ldots(2n-1)}{2^n}\sqrt{\pi}$$

$\therefore$