$$\int_0^\infty e^{-x^2} dx$$

Genelleştirilmiş İntegralinin Değerinin Hesaplanması

Abdullah AYDEMİR

$$\mathrm{I} = \int_0^\infty e^{-x^2}\, dx = \int_0^\infty e^{-y^2}\, dy$$

$$\mathrm{I}^2 = \left(\int_0^\infty e^{-x^2}\, dx\right)\left(\int_0^\infty e^{-y^2}\, dy\right)$$

$$\mathrm{I}^2 = \int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)}\, dxdy$$

$$\begin{bmatrix} x = r\ Cos\theta & r^2 = x^2 + y^2 \\ y = r\ Sin\theta & dx\ dy = r\ dr\ d\theta \end{bmatrix}$$

$$\mathrm{I}^2 = \int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{\infty} e^{-r^2}\, rdrd\theta$$

$$\mathrm{I}^2 = \left(\int_{\theta=0}^{\frac{\pi}{2}} d\theta\right)\left(\int_{r=0}^{\infty} re^{-r^2} dr\right)$$

$$\mathrm{I}^2 = \left(\frac{\pi}{2}\right)\left(-\frac{1}{2} e^{-r^2} \Big|_0^\infty\right)$$

$$\mathrm{I}^2 = \left(\frac{\pi}{2}\right)\left(\frac{1}{2}\right) = \frac{\pi}{4}$$

$$\mathrm{I} = \frac{\sqrt{\pi}}{2}$$

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$